

## Taking Model to the Data with Hybrid Methods

HYUNGJU CHA\*

*The Graduate Center, City University of New York*

### ABSTRACT

Stochastically singularity and tensions between RBC (or DSGE) and VAR (or S-VAR), we need to employ the hybrid method, which relaxes the tension between them. In this point of view, the goal of this paper is the same as the goal of the research using S-VAR, I attempts to base on the power of economic theory while keeping up the flexibility of empirical methods using Kalman filter and Bayesian estimation.

JEL Classification: C32; C51; C52; E32; E37

Keywords: DSGE, RBC, Bayesian Estimation.

### INTRODUCTION

In the 1980s, Real Business Cycle (hereafter RBC) model in Kydland and Prescott (1982) paper is typical model to analyze the business cycle. In their paper, especially, comparing simulated moments, standard deviation of variables, and co-movements between variables with actual data moments using Hodrick-Prescott filter is used. After them, this is typical way in RBC literatures.

After RBC model, this standard model is combined with nominal rigidities, kind of incomplete market stuffs. For example, these are Yun (1996), Goodfriend and King (1997), Rotemberg and Woodford (1995, 1997), and McCallum and Nelson (1999). Due to the market frictions, RBC model is renamed as Dynamic Stochastic General Equilibrium (hereafter DSGE) model. This is because both models are still based on Robinson-Crusoe type economy, one-person and two commodities economy.<sup>1</sup>

---

\* Corresponding Author: E-mail: hcha@gc.cuny.edu or chyunju@sjcny.edu

Any remaining errors or omissions rest solely with the author(s) of this paper.

<sup>1</sup> Big difference between RBC and DSGE is 1<sup>st</sup> and 2<sup>nd</sup> Welfare theorem is not satisfied in or Walrasian Equilibrium is not existed in DSGE. I think that the main reason is DSGE is based on Keynesian perspective on economy. In addition, this is reason why DSGE is called New-Keynesian model.

Because DSGE model is coming from the same vein, optimization method and calibration is still useful to analyze the effect of monetary policy on the economy. Of course, there is a difference between RBC and DSGE, that is, DSGE includes multiple shocks in this model.

As pointed out in Ingram *et al.* (1994) and Prescott (1986), Real Business Cycle model has some problem. In RBC model, the workhorse of business cycle phenomena is the productivity shock. According to Prescott (1986), this randomness explains approximately 70% of variation in real GNP. Because single source of randomness is the productivity shock, stochastically singularity<sup>2</sup> problem is arising while DSGE is not. (This is because DSGE includes multiple shocks in the model.)

In addition to this, as Ireland (2004) pointed out, there is the other problem, that is, great tensions between the data and the economics theory. As we know, Vector Autoregressive model is taken from the data while RBC or DSGE model is firmly based on the theory. In other words, calibrating parameter is based on theoretical optimal decisions while parameters in VAR are not<sup>3</sup>.

In order to get subjective time preference,  $\beta$ , for example, RBC Theorist used Euler equation (sometimes called inter-temporal optimal condition),  $u'(c_t) = \beta E_t[u'(c_{t+1})(1 + r_{t+1})]$ . Under steady-state, we can get  $u'(c_s) = \beta u'(c_s)(1 + r_s)$

This implies that  $\beta = \frac{1}{(1 + r_s)}$ . Using this formula, RBC theorist calculated subjective time preference based on steady state annual real rate of interest rate. Generally, they assume that this annual rate is equal to 4 per cent based on quarterly interest rate,  $\beta = 0.99$ . From this example, calibrated parameters are heavily based on theoretical model. However, in this approach, it is too stylized to be taken from the data. Therefore, it is not easy to implement hypothesis testing or forecasting.

In contrast with this, in case of VAR or S-VAR, there is no restriction to get  $\beta$ . In other words, put simply, quarterly interest rate data and consumption data could be used to estimate  $\beta$  even with Euler equation as mentioned above. Therefore, this approach is flexible rather than above calculation, that is, it is easy to estimate or forecast. Even with this strength, because its theoretical basis is loose, it is often fail to explain the theory. For example, as point out in Carlstrom *et al.* (2009), Choleski identification is used to indentify VAR in analyzing monetary policy shock in standard Dynamic New Keynesian model. However, if this identification is

<sup>2</sup> Stochastically singularity is “more generally, any model with fewer exogenous unobservable variables than endogenous variables is in this sense” (Ingram *et al.*, 1994, p. 419) Therefore, when this is happened, under-identification problem arise in terms of econometrics.

<sup>3</sup> Of course, Structural VAR is based on the theory. But, I think that the main reason for S-VAR to use the theory is to decompose the variance. Therefore S-VAR is still flexible relative to the RBC or DSGE model.

imposed on, then this identification implies that funds rate, monetary policy shock, does not have contemporaneous shock even though theoretical model assumes that Taylor rule has contemporaneous effects on the economy.

In sum, due to these two reasons, stochastically singularity and tensions between RBC (or DSGE) and VAR (or S-VAR), we need to employ the hybrid method, which relaxes the tension between them, that is, the actual data is used to estimate its parameter based on theoretical optimal condition by incorporating optimal conditions in the model into state-space like Kalman filter for using maximum likelihood estimation or Bayesian estimation. In this point of view, the goal of this paper is the same as the goal of the research using S-VAR<sup>4</sup>, I attempts to base on the power of economic theory while keeping up the flexibility of empirical methods.

### HANSEN (1985) MODEL

In Hansen's (1985) RBC model with indivisible labor, representative agent maximizes expected utility by choosing the consumption  $C_t$  and hours worked  $h_t$ . The expected utility function is

$$U = E \sum_{t=0}^{\infty} \beta^t [\log(C_t) + Bh_t] \quad (1)$$

Where the subjective discount factor,  $\beta$  such that  $0 < \beta < 1$  and disutility parameter in hours worked,  $B = -A(\log(1 - h_0))/h_0$ . In here  $h_0$  is fixed hours worked if agent will work, and  $A$  is parameter such that  $A > 0$ .

The representative agent produces output following Cobb-Douglas production technology.

$$f(\lambda_t, k_t, h_t) = \lambda_t k_t^\theta h_t^{1-\theta} \quad (2)$$

Where labor ( $h_t$ ) and accumulated capital ( $k_t$ ) are input, and  $\lambda_t$  is technology shock which follows stochastic process described below. "Agents are assumed to observe  $\lambda_t$  before making any period  $t$  decisions." (Hansen, 1985, p. 313)

The technology shock follows a first-order Markov process. This follows following law of motion:

$$\lambda_{t+1} = \gamma\lambda + \varepsilon_{t+1} \quad (3)$$

---

<sup>4</sup> For example, Bernanke (1986), Blanchard and Watson (1986), and Sims (1986), "attempt to draw on the power of economic theory while retaining the flexibility of more conventional VAR models." (see Ireland (2004)).

Because produced output is either consumed or invested, following resource constraint must be satisfied.

$$c_t + i_t \leq f(\lambda_t, k_t, h_t) \tag{4}$$

The law of motion for the capital stock is given by

$$k_{t+1} = (1 - \delta)k_t + i_t \quad 0 \leq \delta \leq 1 \tag{5}$$

From this optimal problem, first order conditions are

$$BC_t h_t = (1 - \theta)f(\lambda_t, k_t, h_t) \tag{6}$$

$$1/C_t = \beta E_t \left\{ (1/C_{t+1}) \left[ \theta \left( f(\lambda_t, k_t, h_t) / K_{t+1} \right) + 1 - \delta \right] \right\} \tag{7}$$

### SIMULATED MOMENTS FROM HANSEN’S (1985) MODEL

In order for me to estimate hybrid model, Maximum Likelihood Estimation and Bayesian Estimation using Kalman Filter, I need to program the Hansen’s model before doing these to check whether there is something wrong in my coding. In table 1, standard deviation is in column (a), and correlation with output is in column (b).

**Table 1** Comparison of results<sup>6</sup>

Series	Hansen’s (1985) Result		Simulation Result	
	(a)	(b)	(a)	(b)
Output	1.76 (0.21)	1.00 (0.00)	1.8038	1.0000
Consumption	0.51 (0.08)	0.87 (0.04)	0.5242	0.8690
Investment	5.71(0.70)	0.99 (0.00)	5.7632	0.9914
Capital Stock	0.47 (0.10)	0.05 (0.07)	0.5019	0.3546
Hours	1.35 (0.16)	0.98 (0.01)	1.3730	0.9820
Productivity	0.50 (0.07)	0.87 (0.03)	0.9280	0.9999

<sup>5</sup> Typical Real Business cycle model assumes “Time to Build Aggregation,” Kydland and Prescott (1982). But, as in Hansen (1985), I ignore this.

<sup>6</sup> The numbers in parentheses are sample standard deviations of these statistics. Because Dynare do not provide sample standard deviation, I omitted them. But it is not important to check the wrong.

From above table, Hansen's (1985) result is very similar to my result.<sup>7</sup> Therefore, I can empirically estimate both MLE and Bayesian estimation based on my program.

### KALMAN FILTER

“The idea [of Kalman filter] is to express dynamic system in a particular form called the state-space representation. The Kalman filter is an algorithm for sequentially updating linear projection for the system. Among other benefits, this algorithm provides a way to calculate exact finite sample forecasts and the exact likelihood function for Gaussian ARMA processes, to factor matrix auto-covariance-generating functions or spectral densities, and to estimate vector auto-regressions with coefficients that change over time.” (Hamilton, 1994, chapter 13, p. 372)

Therefore RBC or DSGE model also can be written in the state-space forms and the Kalman filter used to estimate the parameters used in RBC or DSGE model. Especially it can also be used to estimate time-varying parameters in a log-linearized optimal conditions around steady state and to obtain MLE of a state-space model.

In order to rewritten optimal conditions into state-space forms, I have to log-linearize. After log-linearization, I can get 5 log-linearized optimal conditions around steady state. For example,

$$\hat{C}_t + \hat{h}_t = \hat{f}_t(\lambda_t, k_t, h_t) \quad (8)$$

and

$$0 = (1/\beta)\hat{C}_t - (1/\beta)E_t\hat{C}_{t+1} + (1/\beta + 1 - \delta)E_t\hat{f}_{t+1} - (1/\beta + 1 - \delta)E_t\hat{k}_{t+1} \quad (9)$$

where  $\hat{f}_t = \ln(f_t/f)$ ,  $\hat{C}_t = \ln(C_t/C)$ ,  $\hat{h}_t = \ln(h_t/h)$  and  $\hat{k}_t = \ln(k_t/k)$ . Based on this, the transformation for Kalman filter is made. The observable vector is

$$f_t = [\ln(y_t/y)\ln(c_t/c)\ln(h_t/h)]' \quad (10)$$

---

<sup>7</sup> The difference among values of the variables is coming from many reasons. I think that one reason is Hansen used Gauss while I used Matlab having different optimization procedure, and the other reason is that Hansen's optimization method is linear quadratic approximation used in Kydland and Prescott (1982) while I used Dynare's optimization method, that is, perturbation method. In addition to this, simulated moments in Hansen's (1985) is calculated based on 100 periods of simulation, whereas, my simulated values are calculated based on 115 periods. But, as pointed out in Hansen (1985), 115 periods are same number of period in U.S. sample

The unobservable vector is

$$x_t = [s_t' u_t'] \quad (11)$$

where  $s_t [\ln(k_t/k) \ln(\lambda_t \lambda)]$  and  $u_t$  is residual vector. This keeps track of the unobserved state variable: capital stock and technology shock.

Residual vector, observed vector and unobserved vector follow each law of motion

$$f_t = C s_t + u_t \quad (12)$$

$$s_t = A s_{t-1} + B \varepsilon_t \quad (13)$$

$$u_t = D u_{t-1} + \xi_t \quad (14)$$

where  $u_t = [u_{yt} u_{ct} u_{ht}]$  and  $\xi_t = [\xi_{yt} \xi_{ct} \xi_{ht}]$  and  $A$ ,  $B$ , and  $C$  are matrices which depends on the model's structure parameters. In addition,

$$D = \begin{bmatrix} d_{yy} & d_{yc} & d_{yh} \\ d_{cy} & d_{cc} & d_{ch} \\ d_{hy} & d_{hc} & d_{hh} \end{bmatrix} \quad (15)$$

Now rewrite empirical model more compactly as

$$x_t = F x_{t-1} + v_t = \begin{bmatrix} A s_{t-1} \\ D u_{t-1} \end{bmatrix} + \begin{bmatrix} B' \varepsilon_t \\ \xi_t \end{bmatrix} \quad (16)$$

and

$$f_t = G x_t = [C I_{3 \times 3}] \begin{bmatrix} s_t \\ u_t \end{bmatrix} \quad (17)$$

for all  $t = 1, 2, \dots, T$  where

$$F = \begin{bmatrix} A & 0_{2 \times 3} \\ 0_{3 \times 2} & D \end{bmatrix}, \quad G = [C \quad I_{3 \times 3}]$$

and 0 and I denote zero and identity matrices.

The serially uncorrelated innovation vector is constructed as

$$v_t = [B' \varepsilon_t \xi_t'] \quad (18)$$

where  $\varepsilon_t$  is the technology shock in RBC model,  $\xi_t$  is the innovation to the residuals. This is because, as mentioned above, stochastically singularity is arisen when there is only one shock, like technology shock. Therefore variance covariance matrix is

$$Ev_t v_t' = Q = \begin{bmatrix} \sigma BB' & 0_{2 \times 3} \\ 0_{3 \times 2} & V \end{bmatrix} \quad (19)$$

and

$$V = \begin{bmatrix} v_y^2 & v_{yc} & v_{yh} \\ v_{cy} & v_c^2 & v_{ch} \\ v_{yh} & v_{ch} & v_h^2 \end{bmatrix}$$

After constructing Kalman filter which is based on theoretical optimal condition, I can estimate RBC or DSGE using actual data to check how much calibration value is close to estimated parameter values. This is reason why this way is called hybrid model. This is because estimation is heavily based on theoretical foundation, especially, log-linearization.

## DATA

In this paper, I use same data as used in Ireland (2004). According to Ireland (2004) “Data for consumption, investment, output and population are taken from the Federal Reserve Bank of St. Louis’ FRED database; data for hours worked come from the Bureau of Labor Statistics’ Establishment Survey. The series are quarterly and run from 1948:1 through 2002:2.” (p.1210) This is because model’s variables are  $Y_t$ ,  $C_t$ ,  $I_t$  and  $h_t$ . But, I extended the data series which are from 1948Q1 to 2004Q2. In addition, according to Ireland (2004), “each series is converted into per-capita terms by dividing by the civilian, non-institutional population, age 16 and over. All data, except for population, are seasonally adjusted. Since the RBC model implies that output, consumption, and investment grows at the common rate  $\eta$  in steady state, the data are automatically detrended as part of the estimation process; they are not filtered in any other way.” (p. 1210) Due to this reason, as following Ireland (2004), I also detrended data by (1) taking log in each series, (2) detrended by Hodrick-Prescott filter, (3) and then take a first-difference in each series. One important point to note is that Ireland’s dataset did not include capital stock variable.

Of course, Ireland’s excluding capital stock data does make sense. This is because it is not easy to measure capital stock quarterly. Cooley and Prescott (1995, p.22) argued that the method to ignore capital is reasonable. This is because “One can interpolate quarterly versions of these, [capital stock], but any procedure for

doing so is essentially arbitrary and may add to the variability of both output and the residuals”.

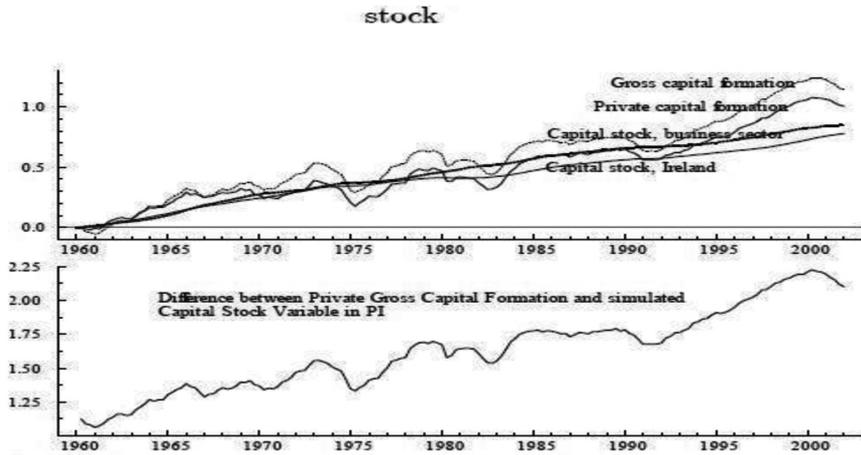
In addition to this, Gomme and Rupert (2007, p. 462) pointed out that “the capital stock data is not well measured. The manifestation of the mis-measurement is that the data are subject to substantial, periodic revision” and “there is also an issue concerning what should be included in the capital stock.” I think that due to these reasons Ireland did not incorporate capital stock data in the estimation.

But excluding the capital stock data has problem. Juselius and Franchi (2007) argued that “Choice of Ireland’s simulated capital stock variable is that it essentially has been designed to conform with the assumed model.” (p. 10) In other words, “capital stock,  $k_t$ , is unobservable, and based on Kalman filter of the RBC model generates a series for capital [stock] assuming that  $(1 - \delta) = 0.975$ .” (p. 9) This means Ireland’s (2004) choice makes capital stock be based on theoretical model. Therefore this does not make sense in this point of view.

Due to this reason, in order to include capital stock variable, Juselius and Franchi (2007) compared three different measurements of capital stock formation, gross capital formation, private capital formation, capital stock in business sector, with the capital stock variable used by Ireland (2004) to include capital stock data. They concluded that “the results suggest that private capital stock formation is more closely related to the simulated TFP than to simulated capital stock and that the linear trend in the simulated capital differs from the trend in the official series.” (p.11) Due to this conclusion, they used private capital formation. (See Picture 1.) I think that this does not seem to be reasonable. In other words, I think that there is the problem in choosing private capital stock formation. This is because choosing data similar to the simulated means that researcher choose the data intentionally, in other words, choosing private capital stock is intentionally to match the theoretical capital stock, that is, there is no reason to choose the private capital.

Due to this reason, I include capital stock which is published in Cleveland Federal Reserve<sup>8</sup>. This dataset is constructed based on Gomme and Rupert (2007) But, problem is that there is 3 types of measurement in capital stock. These capital stock datasets are current cost deflated by consumption deflator (nondurables and services), capital stock data constructed from annual capital data and quarterly investment data (chain-type index converted to real 2000 dollars), and capital stock cumulated from 1947 using investment data and computed depreciation rates. The dataset used in this paper is quarterly, run from 1948:1 to 2004:2 and unit is per capita. Because there is no criterion to select which capital stock dataset is suitable for the estimation, estimation using each dataset is more reliable.

<sup>8</sup> Datasets come from <http://www.clevelandfed.org/research/models/rbc/Index.cfm>



**Figure 1** Three different measurements of capital stock formation compared to the capital stock variable used by Peter Ireland<sup>9</sup>

Moreover, in order to dataset to be consistency, I constructed new dataset for this paper instead of using Ireland's (2004) dataset.

Before estimation, it is important to check whether the datasets are differencing stationary or trend stationary. As pointed out in Nelson and Plosser (1982), because most time series data can not reject the hypothesis that these time series data are non-stationary stochastic processes with no tendency to return to a trend line. Therefore, if the datasets used in this paper has a unit root, it is not reasonable to use Kalman filter. In other words, employing VAR or VECM model would be more suitable approach relative to Kalman filter estimation, or I need to assume that the dataset are trend stationary.

After Augmented Dickey-Fuller and Phillips-Perron testing, I concluded that all data series except worked hours series do not have unit root. To my knowledge, there is no reason to employ VAR or VECM when series do not have unit root. This results makes Kalman filter more robust estimation relative to VAR or VECM. Note that the null hypothesis is that the series has a unit root. Therefore, Due to this, I used Hodrick-Prescott filter instead of differencing as in Ireland (2004).

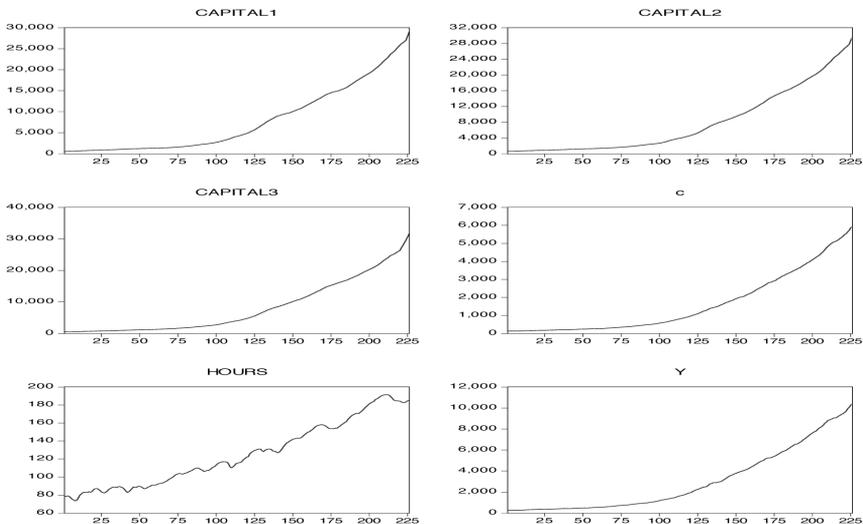
<sup>9</sup> This graph comes from p. 11 in Juselius and Franchi (2007).

**Table 2** Unit root test results

Variables	Augmented Dickey-Fuller			Phillips-Perron		
	Intercept	Intercept and trend	None	Intercept	Intercept and Trend	None
Output	4.42***	2.94***	3.41***	12.43***	2.49***	17.96***
Worked hours	0.83	-2.29	3.59***	0.40	-2.52	3.94***
Consumption	4.59***	4.43***	2.40***	16.92***	5.12***	23.43***
Capital 1	1.73***	1.73***	1.77***	12.01***	3.69***	17.67***
Capital 2	1.16***	1.43***	1.45***	14.44***	4.75***	20.73***
Capital 3	1.38***	1.52***	1.44***	10.13***	3.68***	14.55***

### ESTIMATION USING VAR

Before estimation using Kalman or Bayesian, I estimated above model using VAR model. One reason for this is showing the tension between economic theory and econometric method and how much both are fit to each other. . In order to estimate, I checked whether there is structure break in the dataset. In the dataset, we can see that there is no apparent structure break in the data set. To my knowledge, the main reason of no structure break is that this dataset is partially raw data. In other words, this is because the dataset is generated by Gomme and Rupert (2007).



**Figure 2** Time trend in each variable.

Because based model is heavily based on economic theory, VAR model is also restricted by economic theory, that is, have to be S-VAR. In details, all optimal conditions are used for constructing S-VAR. But, in case of S-VAR system can not incorporate these equations into system equations, one assumption is imposed on the system. The assumptions is that I impose the law of motion of output,  $y_t = (1 + \gamma)y_{t-1}$ <sup>10</sup>. After imposing this restriction, I can get the following system equations.

$$\begin{bmatrix} c_t \\ k_t \\ h_t \\ y_t \end{bmatrix} = \begin{bmatrix} \beta(1+r) & 0 & 0 & 0 \\ -1 & (1-\delta) & 0 & 1 \\ \frac{\beta(1+r)}{B} & 0 & 0 & (1-\theta)(1+\gamma) \\ 0 & 0 & 0 & (1+\gamma) \end{bmatrix} \cdot \begin{bmatrix} c_{t-1} \\ k_{t-1} \\ h_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{c,t} \\ \varepsilon_{k,t} \\ \varepsilon_{h,t} \\ \varepsilon_{y,t} \end{bmatrix} \quad (20)$$

In order to construct system equation, all system equations are based on economic theory. One point I need to note is that because all variables are stationary, except that hours worked, there is no difference between OLS estimation and VAR estimation (or S-VAR estimation).

For constructing the system, system is constructed as follows: the first equation is

$$\frac{1}{c_t} = \beta(1+r)E_t \frac{1}{c_{t+1}} \Rightarrow E_t c_{t+1} = \beta(1+r)c_t \quad (21)$$

This implies that  $c_t = \beta(1+r)c_{t-1} + \varepsilon_{c,t}$ . The second equation is based on law of motion of capital, that is,

$$k_{t+1} = (1-\delta)k_t + i_t \Rightarrow k_t = (1-\delta)k_{t-1} + i_{t-1} \quad (22)$$

Because of resource constraint,

$$c_t + i_t = y_t \Rightarrow i_t = y_t - c_t \quad (23)$$

Therefore, we can get  $k_t = (1-\delta)k_{t-1} + y_{t-1} - c_{t-1} + \varepsilon_{k,t}$ . In this way, we can also get the equation for estimation for worked hours.

From estimation, empirical estimation shows that  $\beta(1+r) = 1.015$ ,  $(1-\delta) = 0.8418$ ,  $\frac{\beta(1+r)}{B} = -0.5076$ ,  $(1-\theta)(1+\gamma) = 0.3035$  and  $(1+\gamma) = 1.014$  in case of Capital 1 variable. Note that I impose the restrictions,

<sup>10</sup>This is simply based on assumption that change in output, the growth, is coming from technology shock.

-1 and +1, when I estimate  $(1 - \delta)$  as shown above. Therefore,  $\beta = 1.005$  if quarterly interest rate as in Hansen (1985) is equal to 1%,  $\delta = 0.1582$ ,  $A = 1.9996$  and  $\theta = 0.7$ . In this way, I estimate other two capital variables. Note that, in case of fixed amount of worked hours, this can not be estimated. The result is as follow:

<b>Variables</b>	<b>Calibration</b>	<b>Capital 1</b>	<b>Capital 2</b>	<b>Capital 3</b>
$B$ (Parameter in utility function)	2	1.9996	1.9996	1.9996
$\beta$ (Subjective time preference)	0.99	1.005	1.005	1.005
$\delta$ (Depreciation rate)	0.025	0.1582	0.1557	0.1495
$\theta$ (Capital share)	0.36	0.7	0.7	0.7
$\gamma$ (First order coefficient for technology)	0.95	0.014	0.014	0.014

In this result, only difference among capitals is depreciation rate. In this estimation, we can see that calibrated parameter values are not satisfied by empirical results.

### ESTIMATION USING MAXIMUM LIKELIHOOD ESTIMATION

Using above data set, I estimate each parameters in the model. From estimated result, we can see that there is some difference between calibrated values and estimated values.

<b>Variables</b>	<b>Calibration</b>	<b>Capital 1</b>	<b>Capital 2</b>	<b>Capital 3</b>
$A$ (Parameter in utility function)	2	2.049	2.11	2.065
$\beta$ (Subjective time preference)	0.99	0.6094	0.9373	0.6552
$\delta$ (Depreciation rate)	0.025	0.2672	0	0.3106
$\theta$ (Capital share)	0.36	0.4512	0.4696	0.6278
$h_0$ (Fixed amount of worked hours)	0.53	0.6332	0.7016	0.643
$\gamma$ (First order coefficient for technology)	0.95	0.9988	0.9994	0.9636

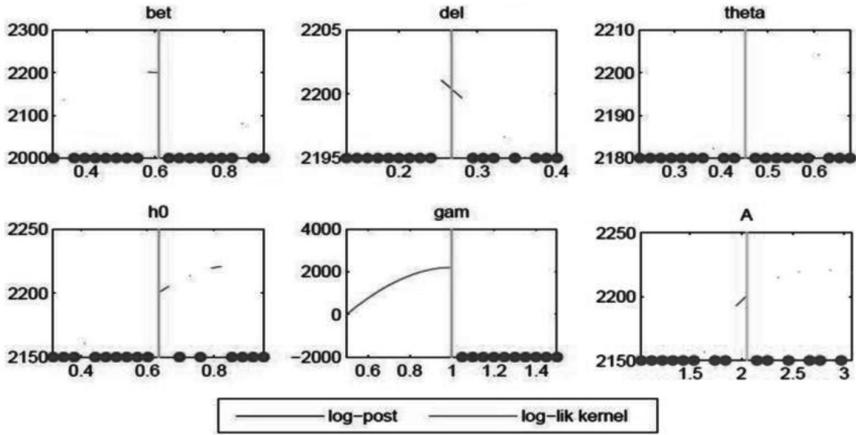


Figure 3 Estimation result using Capital 1 dataset

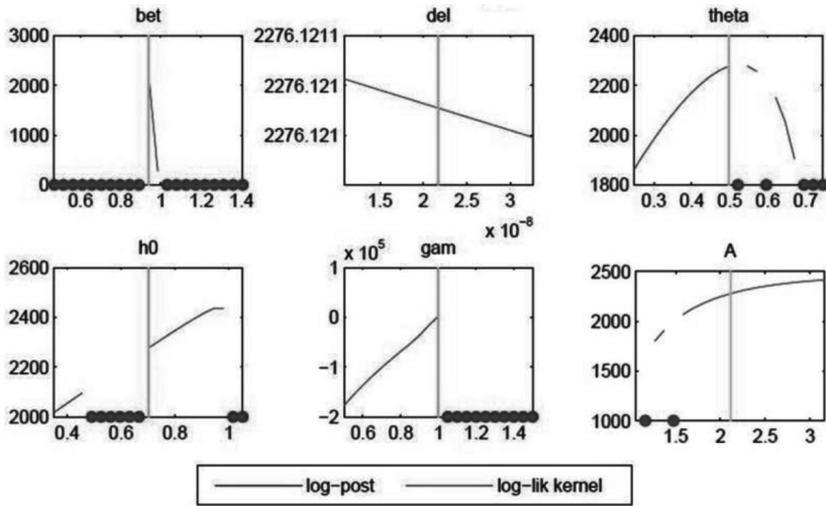


Figure 4 Estimation using Capital 2 dataset

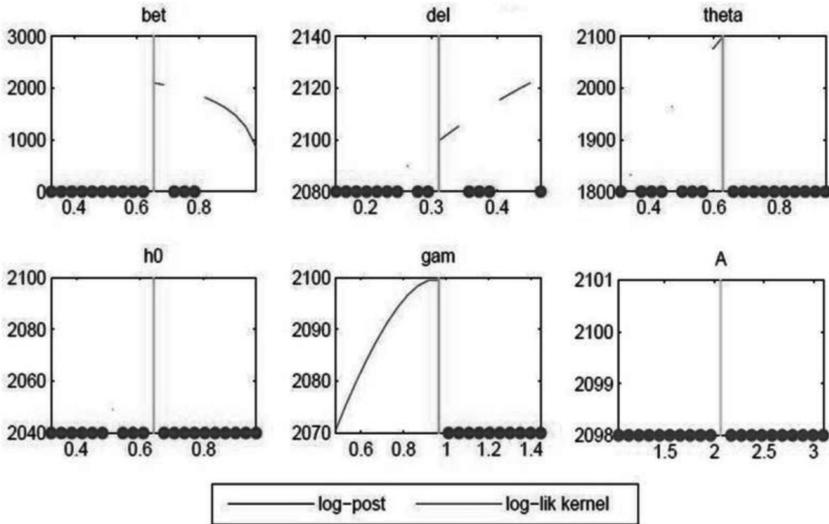


Figure 5 Estimation using Capital 3 dataset

### ESTIMATION USING BAYESIAN

In case of Bayesian estimation, there is no criterion to choose prior density function. Therefore I just followed examples in Dynare User Guide. In this estimation, we can see that there is still difference between them while Bayesian estimated values are similar to each other.

This would be because Bayesian estimation is based on prior density, in other words, estimated values depend on the what prior density function I choose.

Variables	Calibration	Capital 1	Capital 2	Capital 3
$A$ (Parameter in utility function)	2	2.003	2.003	2.002
$\beta$ (Subjective time preference)	0.99	0.9769	0.9769	0.9773
$\delta$ (Depreciation rate)	0.025	0.01043	0.01034	0.0103
$\theta$ (Capital share)	0.36	0.3563	0.3563	0.3535
$h_0$ (Fixed amount of worked hours)	0.53	0.5364	0.5364	0.5345
$\gamma$ (First order coefficient for technology)	0.95	1	1	1

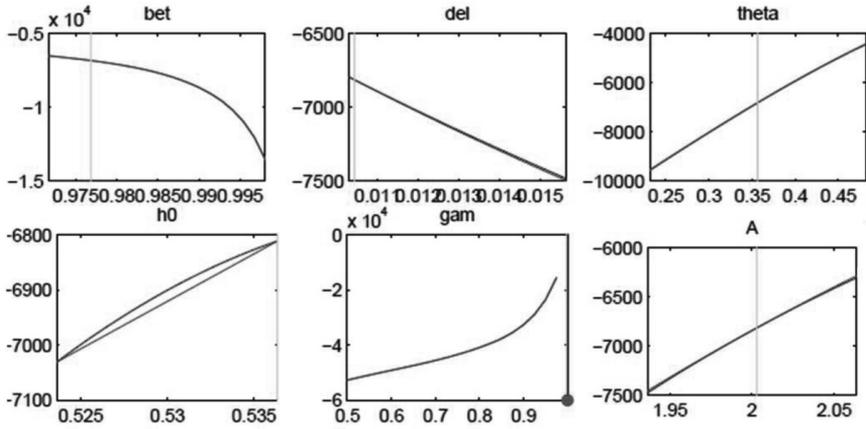


Figure 6 Bayesian Estimation using Capital 1 dataset

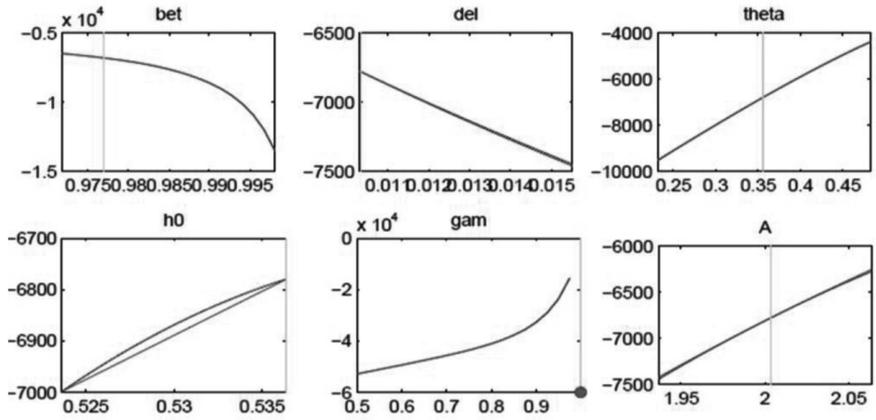


Figure 7 Bayesian Estimation using Capital 2 dataset

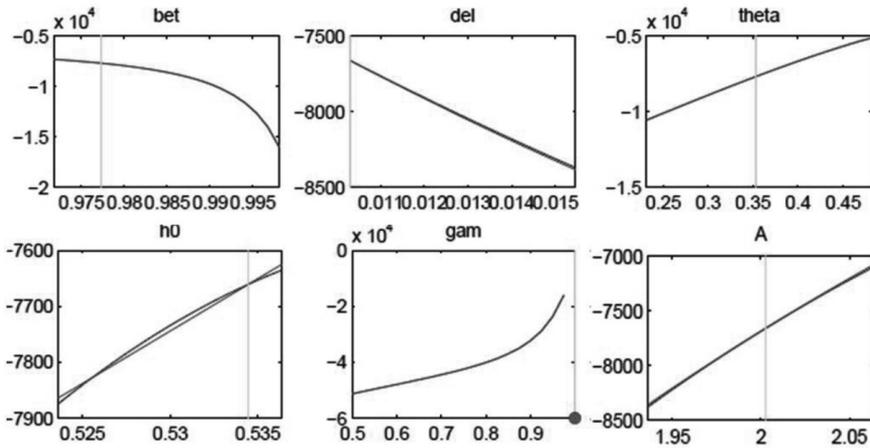


Figure 8 Bayesian Estimation using Capital 3 dataset

## CONCLUSION

Main task of this paper is to decrease the tension between two distinct approaches, empirical estimation and DSGE (or RBC.) Because both approach have their own strength and weakness. Therefore if we find the some point where both, flexibility of VAR and power of the DSGE, are satisfied, we can check the theory can be supported by the data and robustness of the model easily.

In this paper, I estimated the parameters using VAR and theory based Maximum likelihood and Bayesian estimation. The estimation shows that there is little difference between them. But, this difference is still rely on which estimation method we use. Therefore trying to decrease tension between empirical estimation and theoretical model is needed for future studies.

## REFERENCES

- Bernanke, B. S. (1986) Alternative Explanations of the Money-Income Correlation, *Carnegie-Rochester Conference Series on Public Policy*, **25**, 49-99.
- Blanchard, O. J. and Watson, M. W. (1986) Are Business Cycles all alike?, in *The American Business Cycle: Continuity and Change* (Ed.) R. J. Gordon, University of Chicago Press, Chicago, IL, pp. 123-179.
- Carlstrom, Charles T., Fuerst, Timothy S., and Paustian, Matthias (2009) Monetary Policy Shocks, Choleski Identification, and DNK Models. *Journal of Monetary Economics*, **56**, 1014-1021.

- Cooley, T. F and Prescott, E. C. (1995) Economic Growth and Business Cycle, in *Frontier of Business Cycle Research* (Ed.) T. F. Cooley, Princeton University Press, Princeton, pp. 243-293.
- Gomme, P. and Rupert, P. (2007) Theory, Measurement and Calibration of Macroeconomic Models, *Journal of Monetary Economics*, **54(2)**, 460-497.
- Goodfriend, M. and King, R. G. (1997) *The New Neoclassical Synthesis and the Role of Monetary Policy*, MIT Press: NBER Macroeconomics Annual Cambridge, MA, pp. 231-283.
- Hamilton, J. D. (1994) *Time Series Analysis*. Princeton University, Princeton, NJ.
- Hansen, G. D. (1985) Indivisible Labor and the Business Cycle, *Journal of Monetary Economics*, **16**, 309-327.
- Ingram, B. F., Kocherlakota, N. R. and Savin, N. E. (1994) Explaining Business Cycles: A Multiple-Shock Approach, *Journal of Monetary Economics*, **34**, 415-428.
- Ireland, P. N. (2004) A Method for Taking Model to the Data, *Journal of Economic Dynamics & Control*, **28**, 1205-2226.
- Juselius, K. and Franchi, M. (2007) *Taking a DSGE Model to the Data Meaningfully*, *Economics - The Open-Access, Open-Assessment E-Journal*, Kiel Institute for the World Economy, **1(4)**, 1-38.
- Kydland, F. E. and Prescott, E. C. (1982) Time to Build and Aggregate Fluctuations, *Econometrica*, **50(6)**, 1345-1370.
- McCallum, B. T. and E. Nelson (1999) An Optimizing IS-LM Specification for Monetary Policy and Business Cycle Analysis, *Journal of Money, Credit, and Banking*, **31(3)**, part 1, 296-316.
- Nelson, C. R. and Plosser, C. I. (1982) Trends and Random Walks in Macroeconomic Time Series, *Journal of Monetary Economics*, **10**, 139-162.
- Prescott, E. C. (1986) Theory ahead of business cycle measurement, *Quarterly Review, Federal Reserve Bank of Minneapolis*, **Fall issue**, 9-22.
- Rotemberg, J. J. and M. Woodford (1995) Dynamic General Equilibrium Models with Imperfectly Competitive Product Markets, in *Frontier of Business Cycle Research* (Ed.) T. F. Cooley, Princeton University Press, Princeton, pp. 243-293.
- \_\_\_\_\_ (1997) An Optimizing-Based Econometric Model for the Evaluation of Monetary Policy, NBER Macroeconomics Annual, MIT Press: Cambridge, MA, pp. 297-346.
- Sims, C. A. (1986) Are Forecasting Models Usable For Policy Analysis? *Federal Reserve Bank of Minneapolis Quarterly Review*, **10**, 2-16.
- Yun, T. (1996) Nominal Rigidity, Money Supply Endogeneity, and Business Cycle, *Journal of Monetary Economics*, **37(2)**, 345-370.